ON THE MEASUREMENT OF A PHYSICAL QUANTITY WHOSE MAGNITUDE IS INFLUENCED BY PRIMARY CAUSES BEYOND THE CONTROL OF THE OBSERVER AND ON THE METHOD OF DETERMINING THE RELATION BETWEEN TWO SUCH QUANTITIES

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Read before the Academy, April 25, 1922

In order to relate the physical and chemical properties of carbon to the microphonic properties of this material, it has been found necessary to make measurements on physical quantities which are influenced by primary causes beyond the control of the observer. In many problems of the physical and engineering sciences it is possible for the observer to control within narrow limits, the causes of variation of a quantity while it is being subjected to measurement. Certain problems arise, however, in these sciences as in the field of economics and biology wherein it is impossible to do this. In general let X_1 and X_2 represent two quantities related to others $U_1, U_2, \ldots U_a; V_1, V_2, \ldots V_b; W_1, W_2 \ldots W_b$ in the following way:

$$X_1 = F_1 (U_1, U_2, \ldots U_a, V_1, V_2, \ldots V_b) \qquad (1)$$

$$X_2 = F_2 (U_1, U_2, \ldots U_a, W_1, W_2, \ldots W_b) \qquad (2)$$

where F_1 and F_2 represent unknown functional relations. In most physical experiments it is possible to hold the U's, V's, and W's, practically constant while a measurement is being made on either X_1 or X_2. In the last analysis, however, the U's, V's, can never be held constant and in many cases, particularly where these symbols represent molecular phenomena, the variations about their mean values may become quite large.

Thus, if X_1 and X_2 represent two microphonic characteristics of granular carbon which are functions of the physical and chemical properties represented by the U's, V's, and W's, it has been found necessary to apply certain statistical criteria to determine the nature of the cause complex controlling a single quantity such as either X_1 or X_2 and also to determine quantitatively the degree of relation existing between the two microphonic properties. As a result of such a study involving an analysis of thousands of measurements of the above type, certain conclusions which are of general interest have been reached in respect to the practical application of statistical methods in connection with physical measurements of this character.

In order to study the nature of the complex of causes controlling a single quantity such as X_1, one of the first problems is to determine whether or not the causes represented by the U's, and V's satisfy the following conditions: (1) That all of the causes, n in number, are effective at the time of each observation, (2) that the probability, p, that a cause will produce a positive effect is the same for all of the causes. (3) That the probability, p, remains the same for all of the observations, (4) that the effect, Δx, of a single cause is the same for all of the causes.

If these conditions are fulfilled, the distribution in X_1 can be represented by the successive terms of the expansion $(p + q)^n$ where the ordinates are separated at intervals of $2\,\Delta x$. For most of the problems it has been found convenient to compare the observed distribution with the theoretical distribution consistent with the above random conditions. In general, the following procedure has been followed: For each observed distribution two factors $k = \dfrac{\mu_3}{\mu_2^{3/2}}$ and $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$ have been calculated and compared with similar factors consistent with the above mentioned binomial expansion, where the first four corrected moments of the observed distribution about the mean are represented by the symbols μ_1, μ_2, μ_3, and μ_4. It will be noticed that k and β_2 are independent of the units used in making the measurements.

For a symmetrical distribution k is always 0 and β_2 may vary between 1 corresponding to $n = 1$ and 3 corresponding to $n = \infty$. Irrespective of the values of p and q, it should be noted that β_2 does not increase beyond 4 for 10,000 causes and even for 100 causes the value of 4 is not exceeded except for conditions in which the skewness k is very large. In general large values of β_2 indicate either high skewness and few causes or that the complex of causes does not follow the random conditions consistent with the expansion of $(p + q)^n$.

As a result of variations due to sampling the standard deviations, σ_k and σ_{β_2} of k and β_2 are given in terms of the number of observations, s, by the following expressions $\sigma_k = \sqrt{\dfrac{6}{s}}$ and $\sigma_{\beta_2} = \sqrt{\dfrac{24}{s}}$ it being assumed that the distribution is practically normal. If, in practice, the values of k and β_2 are found to differ from 0 and 3 respectively by more than three times the standard deviations of k and β_2, it is practically certain that the distribution is either unsymmetrical or that the causes do not act at random as defined above. The method of application of these criteria depends in general upon whether or not the observed distribution is one involving attributes or variables.

In general for the case of attributes the procedure is as follows: The factors k and β_2 are calculated to determine whether or not they are signif-

icantly different from 0 and 3. If k and β_2 are different from 0 and 3, the values of p, q, and n may be calculated since $pn = \bar{x}$ and $pqn = \mu_2$ where $\bar{x}$ represents the average of the distribution, and a study of these values will indicate the probable reason for such variations in k and β_2 provided the distribution follows the random laws. If the observed distribution is consistent with the calculated values of p, q, and n, the "probability of fit"[1] between the observed distribution and the theoretical distribution representing the expansion of $(p + q)^n$ should be high.

In the case that the U's, V's and W's are variables, we start with unknown values of p, q and n, and in addition have the unknown value of ΔX which represents the effect of a single cause. It is obviously impossible to determine the values of p, q and n as in the case of attributes for among other reasons we cannot determine the origin of the distribution. In general, however, we may make use of the factors, k and β_2, since these are independent of the unit in which the measurements are made. Furthermore, we may make use of the criterion, establishing the "goodness of fit" between the observed distribution and a theoretical one corresponding to the various degrees of approximation[2] to the normal law.

To return to equations (1) and (2) it is evident that, if none of the causes are common, the correlation between simultaneous measurements of X_1 and X_2 will be practically zero. Furthermore, as the number of causes that are common increases we should expect an increase in the correlation coefficient under certain limiting conditions. Though it may not be possible to determine the number of causes that influence a variable quantity, such as either X_1 or X_2, it has been found possible, however, to determine the approximate value for the ratio, of the number of common causes to the total number of causes influencing either variable. To show this for the general case assumed in equations (1) and (2) let us start with the definition of the correlation coefficient r,

$$r = \frac{\Sigma x_1 x_2}{N \sigma_{x_1} \sigma_{x_2}} \tag{3}$$

where x_1 and x_2 represent the deviations from the mean values of X_1 and X_2 and σ_{x_1} and σ_{x_2} represent the standard deviations in these quantities and N represents the total number of observations made. Let us assume independence between the primary causes. If we expand equations (1) and (2) by Taylor's theorem and neglect thes econd and higher powers of the variations of the primary causes we can obtain values of the deviations x_1 and x_2 and of the standard deviations σ_{x_1} and σ_{x_2}, which, when substituted in equation (3) reduce to the simple expression

$$r = \frac{a}{a + b} \tag{4}$$

providing we add the further conditions that the standard deviations in all of the primary causes are equal and that the effects of all of the primary causes are equal.   These conditions appear at first very limited, and in a practical case it is necessary at least qualitatively to check the assumptions underlying equation (4), but in general it has been found that a significant deviation in r may be taken to indicate a variation in the effects of one or more groups of primary causes, upon one of the variables and not upon the other.

A more complete account of this investigation giving experimental data, will be published elsewhere.

[1] Pearson, K. "Tables for Statisticians and Biometricians" pp. (26–28) and *Phil. Mag.*, Ser. 5, **1**, 1900.

[2] Edgeworth, F. Y., *Camb. Phil. Trans.*, **20**, 1904 (36–65) and (113–141).

ELECTRONIC BOMBARDMENT OF NICKEL

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Studies of the secondary (emergent) electrons produced by electronic bombardment of metal surfaces have previously been made by various experimenters.[1]   However, the fact that most of the results were obtained previous to the development of modern high vacuum technique combined with the failure of the small amount of recent work to agree with these older results, leaves the important questions of this problem still un-answered.   It is well known that the number of secondary electrons de-pends upon the velocity of the primary (incident) beam, that more elec-trons leave the surface than strike it if the primary velocity is great enough, but the following characteristics of the various metals are still not definitely determined: (1) the magnitude of the secondary electron current as a function of the primary velocity; (2) the velocity distribution of the secondary electrons for any given primary velocity.

The present investigation is an attempt to obtain these characteristics for nickel.   The type of apparatus employed was such that a primary electron stream, of uniform velocity and cross-section, could be made either to impinge on a nickel target or to pass into an absorbing Faraday cylinder. This was accomplished by moving the target, by means of a magnetic control, over a hole in the Faraday cylinder into the path of the primary stream.   The secondary electrons which left the target were received by a conducting cylinder.   This arrangement made it possible to move the